

Halving to the Atom: How Many Cuts to the Bottom?

Investigation question: How many times would you have to cut a one-meter strip in half to reach the width of a single atom, and why can your scissors never get there?

Physical model

Groups of 3

1 session

Setup 5 · Activity 35 · Cleanup 5 min

MS-PS1-1 · practice

Scientific purpose

Students use a paper strip as a physical model of repeated halving, measure and graph how the length shrinks, discover that scissors fail after about ten cuts, and then extend the pattern with arithmetic to estimate the number of halvings that would reach the picometer scale. The lab makes the size of an atom a number students derived themselves rather than a fact they were told.

Why this helps students with this chapter

Students hear that atoms are tiny but have no feel for how tiny. Halving a real strip, watching the numbers fall, hitting the physical limit of scissors and then continuing the pattern on paper turns the word picometer into a quantity they computed. The failed cuts also surface the misconception that halving can go on forever.

Standards connections

MS-PS1-1 (practice, the scale, proportion and quantity foundation for atomic models): Students build a quantitative sense of the atomic scale and of the atom as the smallest unit of an element, which they need before they can develop models of molecules and extended structures in Lesson 2.

Roles (rotate each session)

- **Cutter:** Folds the strip exactly in half, creases it, and cuts along the crease. Keeps one half and sets the other aside.
- **Measurer:** Measures the kept piece with the meter stick or mm ruler, calls out the value in centimeters and in millimeters.
- **Recorder and grapher:** Writes each measurement in the table, checks it against the predicted value, and plots the points on the graph.

Materials

Item	Quantity	Per	Note
Receipt or adding-machine paper tape, cut to exactly 100 cm	1 strip	group	A 100 cm strip of masking tape stuck to wax paper also works.
Scissors	1 pair	group	

Meter stick	1	group	
Ruler marked in millimeters	1	group	
Hand lens or phone camera zoom	1	group	For measuring the last few pieces.
Graph paper and the recording sheet	1 set	student	
Calculator	1	group	
Small envelope or zip bag	1	group	To keep the cut pieces in order for the debrief.

Estimated cost: 4 to 6 per class (an estimate; Estimate. One roll of adding-machine paper (about \$3) supplies more than 30 strips; scissors, rulers and graph paper are standard classroom stock.)

Safety

- Scissors only: cut away from fingers, pass them handle first, and keep them closed on the desk when not in use.
- Tiny paper scraps are a slip hazard on hard floors; collect them in the envelope as you go.
- No hazards beyond ordinary classroom tools; no chemicals or heat are used.

Teacher preparation and setup

1. Cut one 100 cm strip per group ahead of time and check each with a meter stick; a strip that starts at 98 cm throws off the predicted column.
2. Print the recording sheet and have graph paper ready with the axes already labeled for younger or struggling groups.
3. Do one run yourself so you know where your paper stops being cuttable; most tapes fail between the ninth and eleventh halving.
4. Write the unit chain on the board before class: $1\text{ m} = 100\text{ cm} = 1,000\text{ mm} = 1,000,000\text{ micrometers} = 1,000,000,000\text{ nm} = 1,000,000,000,000\text{ pm}$.

Investigation design

Independent variable	Number of halvings (0, 1, 2, ...)
Dependent variable	Length of the kept piece, in centimeters and millimeters
Controlled	Starting length of 100 cm; Fold-and-cut method (always fold end to end, cut on the crease); Same measuring tools for every group; Always keep one of the two halves, never a smaller fragment
Trials	One run per group; the class compares where each group's scissors failed and pools the results on the board.
Comparison	Predicted length (pure halving) versus measured length at each step.

Student procedure

1. Measure your strip and record its length as halving number 0. It should be 100 cm. Before any cutting, fill in the predicted length column for halvings 1 through 10 by dividing the previous prediction by 2.
2. Fold the strip exactly in half so the ends line up, crease it, and cut along the crease. Keep one half; put the other half in the envelope with a small '1' written on it.
3. Measure the kept half in centimeters (and in millimeters once it is shorter than 10 cm). Record the measured length next to the prediction for halving 1.
4. Repeat: fold the kept piece in half, cut, keep one half, measure, record. Number each discarded piece and save it so the sequence can be lined up later.
5. Keep going until the group agrees the piece can no longer be folded and cut in half accurately. Record the halving number where you stopped and write one sentence about what went wrong.
6. Plot measured length (cm) on the vertical axis against halving number on the horizontal axis. Connect the points with a smooth curve, not straight segments.
7. Lay the saved pieces out in order from largest to smallest on the desk. Look at the pattern and compare it with the shape of your graph.
8. Now continue the pattern with arithmetic instead of scissors: use the fact that ten halvings divide a length by about 1,000 to work out how many halvings take 1 m down to 1 mm, then to 1 micrometer, then to 1 nanometer, and finally to about 200 picometers, the width of a typical atom.
9. Answer the analysis questions individually, then compare answers within the group.

Expected observations

- Measured lengths track the predictions closely for the first six or seven halvings (100, 50, 25, 12.5, 6.3, 3.1, 1.6 cm) and then drift as the pieces get hard to fold and measure.
- Most groups stop between the ninth halving (about 2 mm) and the eleventh (about 0.5 mm); the paper tears or the fold is wider than the piece.
- The graph is a steep curve that drops by half each step and hugs the axis after about halving 7, so most of the graph's height is used in the first three points.

Likely difficulties

- Students fold off-center and the halves differ; insist on lining up the ends before creasing, and have the measurer check both halves at the first few cuts.
- Reading a 3 mm piece on a ruler is hard; the hand lens helps, and students may report the fold width instead of the length.
- The arithmetic extension confuses students who have not met powers of ten. The 'ten halvings is about a thousand' shortcut keeps it in whole steps of the unit chain.
- Some students will insist the strip could be halved forever with a better tool; the debrief and q7 address this directly.

Structured debrief

1. Pool every group's stopping point on the board. Ask why the numbers cluster around 9 to 11 even though groups had different scissors and hands. (Paper is made of fibers roughly 20 to 40 micrometers wide; below about a millimeter the piece tears instead of cutting.)
2. Compare the measured and predicted columns. Where did they drift apart and why? Lead students to see that measurement error grows as the piece shrinks, which is exactly why scientists needed a new tool, not a sharper pair of scissors, at the atomic scale.
3. Walk the arithmetic together: 10 halvings to 1 mm, 20 to 1 micrometer, 30 to 1 nm, and about 32 to 33 to reach a 200 pm atom. Point out that fewer than 35 cuts separate a meter stick from a single atom, which surprises most students.
4. Close on q7: the halving model shows how small atoms are, but it cannot show what happens when you split one. That is where the model stops matching reality, and it is the reason the atom, not a smaller piece, is the unit of an element.

Recording sheet: what students fill in

Lab 1 recording sheet: Halving to the atom

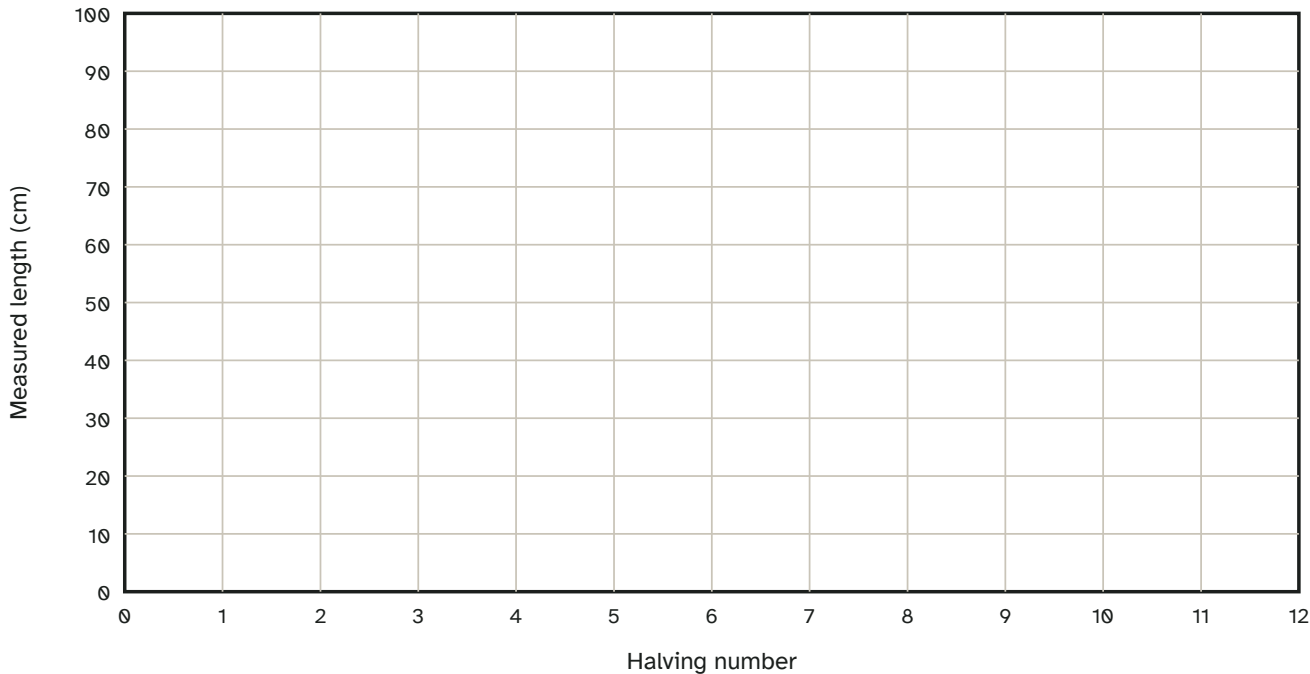
Fill in the predicted column before you cut. After each cut, record what you actually measured. Stop when your group cannot halve the piece accurately.

Halving data

Halving number	Predicted length (cm)	Measured length (cm)	Measured length (mm)	Could you still fold and cut it? (yes/no)
0	100			
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				
11				

Halving number	Predicted length (cm)	Measured length (cm)	Measured length (mm)	Could you still fold and cut it? (yes/no)
12				

Length of the kept piece after each halving



Continuing the pattern with arithmetic

Target length	Same length in meters	Halvings needed from 1 m (estimate)	How you worked it out
1 mm			
1 micrometer (width of a bacterium)			
1 nanometer (1,000 pm)			
About 200 pm (one atom)			

1. At which halving did your group stop, and what exactly made the next cut impossible?

Answer: Group-specific; typically halving 9, 10 or 11. The piece became narrower than the fold, the paper tore instead of cutting, or the ruler could not resolve the length.

Why: Paper fibers are tens of micrometers wide and a fold has a finite width, so cutting fails around a millimeter.

- States the halving number
- Names a specific physical cause (tearing, fold width, measuring limit)

2. Describe the shape of your graph. Why does the curve flatten toward the horizontal axis even though every cut removes half?

Answer: A curve that drops steeply at first and then flattens, hugging the axis. Each cut removes half of what is left, and half of a small number is a small change, so the drops get tiny even though the fraction is always one half.

Why: Exponential decay: equal ratios, shrinking differences. Students need the idea that 'half' is a ratio, not a fixed amount.

- Describes the decreasing, flattening shape
- Explains that half of a small length is a small absolute change

Analysis questions: answer key

3. Ten halvings divide a length by 1,024, which is close to 1,000. Use that fact to estimate how many halvings take a 1 m strip down to 1 mm, and show the calculation.

Answer: About 10 halvings. $1\text{ m} = 1,000\text{ mm}$; ten halvings divide by 1,024, so $1,000\text{ mm} \div 1,024 \approx 0.98\text{ mm}$, essentially 1 mm.

Why: $2^{10} = 1,024$. The approximation 'ten halvings $\approx$ divide by 1,000' is what makes the rest of the chain quick.

Also acceptable: Accept 9.97 or 'just under 10' with the calculation shown.

4. Continue the pattern to a single atom about 200 pm wide. About how many halvings from 1 m does that take? Show your reasoning with the unit chain.

Answer: About 32 to 33 halvings. $1\text{ m} \rightarrow 1\text{ mm}$ is 10 halvings; $1\text{ mm} \rightarrow 1\text{ micrometer}$ is 10 more (20); $1\text{ micrometer} \rightarrow 1\text{ nm}$ is 10 more (30); $1\text{ nm} = 1,000\text{ pm}$, and $1,000\text{ pm} \rightarrow \text{about } 200\text{ pm}$ takes 2 to 3 more halvings ($1,000 \rightarrow 500 \rightarrow 250 \rightarrow 125$). So roughly 32 or 33 halvings. Check: $2^{32} \approx 4.3 \times 10^9$, and $1\text{ m} \div 4.3 \times 10^9 \approx 2.3 \times 10^{-10}\text{ m} = 230\text{ pm}$.

Why: Students should chain three 'thousands' and then halve two or three more times. Any answer from 31 to 34 with a correct chain earns full credit because atom width is a range.

Also acceptable: Accept 30 to 35 with reasoning that uses the unit chain.

5. Your scissors failed after roughly ten halvings, but the arithmetic kept going. What is the difference between a limit of your tools and a limit of the matter itself? Where is each limit in this activity?

Answer: A tool limit is set by the equipment and skill: scissors, paper fibers, the fold width, the ruler. A better tool could push past it. A limit of matter is set by the substance: once you reach one atom of iron, the next split does not give smaller iron. In this activity the tool limit was around halving 10 (about 1 mm); the matter limit, which we only reached by arithmetic, is at about halving 32 or 33.

Why: This is the heart of the lab: separating practical limits from physical ones. Full credit requires both limits located in the sequence.

- Distinguishes tool limit from matter limit
- Locates the tool limit near halving 9-11
- Locates the matter limit near halving 32-33 (the atom)

6. Why do scientists measure atoms in picometers instead of meters or millimeters? Use a number from your table in your answer.

Answer: Because an atom in meters is 0.0000000002 m (2×10^{-10} m) and in millimeters is 0.0000002 mm, which are unreadable. In picometers the same width is about 200 pm, a size you can say and compare. From the table, 1 mm is 1,000,000,000 pm, so millimeters are a billion times too big a unit for atoms.

Why: Scale is about matching the unit to the thing. Any specific number from the unit chain satisfies the prompt.

- Explains the mismatch between unit and object
- Uses a number from the unit chain or table

7. Suppose you could keep halving a strip of pure iron and finally held one iron atom. If you split that atom in half, would you have two smaller pieces of iron? Explain what the halving model can and cannot show about this.

Answer: No. Splitting an iron atom does not give two smaller pieces of iron; the pieces are no longer iron at all. The halving model shows how many halvings it takes to reach one atom and makes its size real, but it cannot show what is inside an atom or what happens when one is split; paper just keeps being paper. That is where the model stops matching the real thing.

Why: The atom is the smallest unit of an element by definition. Students need not know about protons and neutrons; they should recognize the model's boundary.

- States that split pieces are not iron
- Identifies what the halving model shows (size, count of halvings)
- Identifies what it cannot show (inside the atom, the split)

8. If you had started with a strip 1 kilometer long instead of 1 meter, how many more halvings would it take to reach an atom? Explain.

Answer: About 10 more halvings, so roughly 42 to 43 in total. 1 km = 1,000 m, and dividing by 1,000 takes about ten halvings ($2^{10} = 1,024$).

Why: Reinforces that each factor of a thousand costs ten halvings regardless of where you start.

Also acceptable: Accept 'ten more' without the total.

Individual evidence of learning

Each student submits their own completed data table, graph, arithmetic table and written answers to q1 through q8. The graph and q4 are the primary evidence for the scale target; q7 is the evidence for understanding the atom as the limit of an element.

Assessment criteria

Criterion	Proficient looks like
Data and graph	All halvings recorded in both units, predictions computed correctly, points plotted accurately and connected as a smooth decreasing curve.
Scale reasoning	Correctly estimates about 10 halvings to 1 mm and about 32 to 34 to reach an atom, with the unit chain and the 'ten halvings is about a thousand' shortcut shown.

Limits of tools versus limits of matter	Explains that scissors failed because of paper fibers and hand precision, while the atom is a limit of the substance itself, and names where the model stops matching reality.
Use of vocabulary	Uses atom, element, atomic scale and picometer correctly in written answers.

Accessibility adaptations

- Provide strips of wide card and stop at eight halvings for students with limited fine-motor control; the arithmetic extension carries the same learning.
- Offer a pre-filled predicted column and a graph with axes already scaled for students who need more support.
- Allow the measurer role to call out values while a partner records, and accept a spoken explanation for q5 and q7 recorded on a device.
- Provide the unit chain as a printed strip with large text, and offer a multilingual glossary card for halving, length, prediction and scale.

Low-cost alternative

Use strips of newspaper or a 1 m length of masking tape stuck to wax paper, and share one meter stick between two groups. If no mm rulers are available, stop measuring at 1 cm and continue by arithmetic only.

Model limitations

A paper strip is a model of length, not of iron: paper is made of fibers far larger than atoms, so it fails long before the atomic scale. Halving models how small atoms are but cannot model what happens when an atom is split, and the model treats an atom as a sharp-edged object when real atoms have fuzzy edges, which is why 'about 100 to 300 pm' is a range. This activity has not been physically tested by the authors; the stopping point of 9 to 11 halvings is an estimate for typical receipt paper.

Teacher notes

Check q3 and q4 arithmetic for the ten-halvings shortcut rather than a specific decimal. If a group's strip was not exactly 100 cm, their predicted column will be off by a constant factor; the halving count is unaffected, which is itself worth pointing out. Keep the envelope of ordered pieces for the scale-ladder slide; lining them up next to a meter stick is a strong visual.