

Grams per Stroke and the Halfway Rule: Two Datasets

Learning target: I can graph and calculate from data to show that invisible air adds mass in a steady pattern, and test how well the halfway rule predicts the middle element of a triad.

- Data and graphing
- About 40 min
- Individual
- MS-PS1-1 · supports
- data analysis
- independent practice

Standards connections

MS-PS1-1 (supports): Analyzing the mass added by invisible air and the regular pattern of atomic masses gives students quantitative reasons to believe that matter is made of particles with mass, the idea beneath every molecular model in this PE.

Purpose: practice · **Time:** about 40 minutes · **Format:** individual

Answers with reasoning

Part A: How much mass does each pump stroke add?

A student put a rigid basketball on a balance that reads to 0.1 g, pumped it ten strokes at a time, and recorded the mass after each set of strokes. Then she let all the air out and repeated the whole procedure (Trial 2). The ball's size did not change during the test.

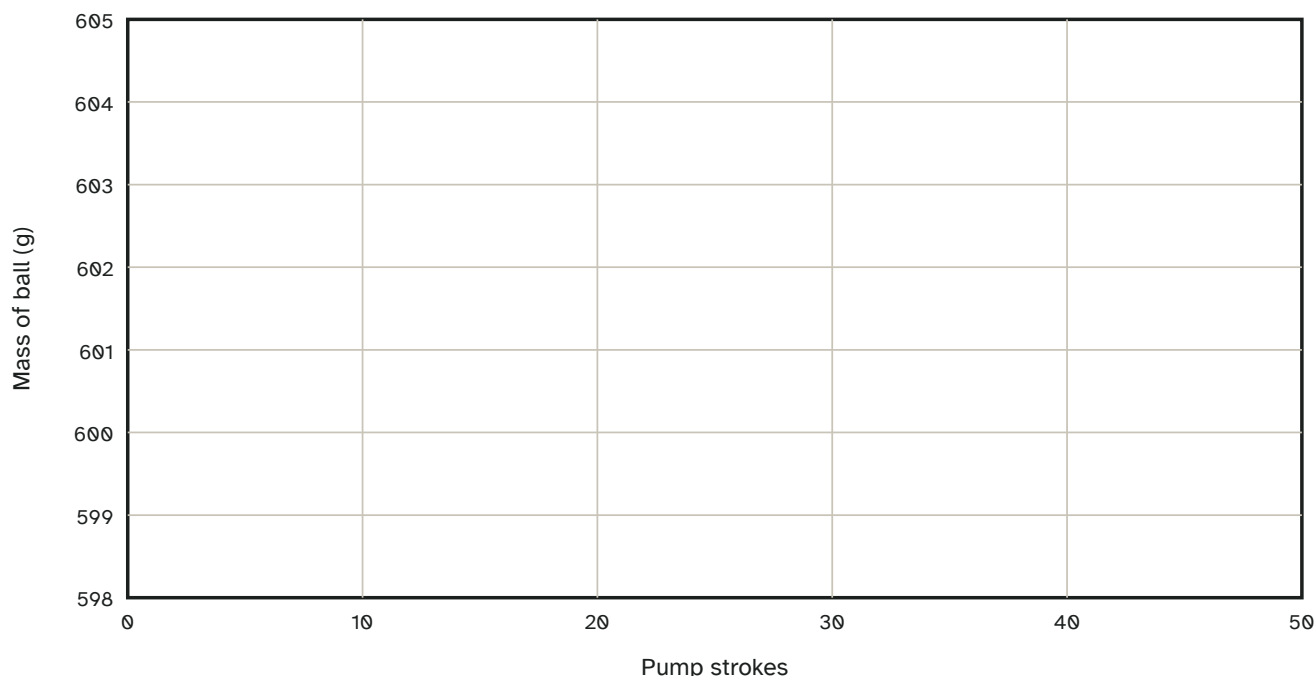
Mass of a rigid basketball after pumping (teaching dataset)

Simulated teaching dataset: the numbers are invented to be realistic, not measured.

Pump strokes	Trial 1 mass (g)	Trial 2 mass (g)
0	598.4	598.4
10	599.6	599.5
20	600.8	600.7
30	601.9	602.0
40	603.1	603.2
50	604.3	604.2

Simulated data for teaching. Realistic for a 7.1 liter ball and a small hand pump.

Mass of the ball versus pump strokes



1. Calculate the total mass gained from 0 to 50 strokes in Trial 1 and in Trial 2. Then find the average of the two gains. Show your work.

Answer: Trial 1: $604.3\text{ g} - 598.4\text{ g} = 5.9\text{ g}$. Trial 2: $604.2\text{ g} - 598.4\text{ g} = 5.8\text{ g}$. Average gain: $(5.9\text{ g} + 5.8\text{ g}) \div 2 = 11.7\text{ g} \div 2 = 5.85\text{ g}$.

Why: Both trials start at the same flat mass, so the gain is simply final minus initial. The two trials agree within 0.1 g, which is the balance's resolution.

Also acceptable: Accept 5.85 g or 5.9 g (if rounded) with the work shown.

2. Using your average gain from q1, calculate the mass added per pump stroke. Round to the nearest hundredth of a gram.

Answer: $5.85\text{ g} \div 50\text{ strokes} = 0.117\text{ g per stroke} \approx 0.12\text{ g per stroke}$.

Why: Dividing the average total gain by the number of strokes gives the rate. The unit must be grams per stroke.

Also acceptable: Accept 0.12 g/stroke; accept 0.118 if the student used 5.9 g.

3. Describe the shape of your graph. What does the shape tell you about how much air each stroke adds?

Answer: The points lie almost exactly on a straight line that rises to the right. A straight line means each stroke adds about the same amount of mass (about 0.12 g); the amount added does not get bigger or smaller as the ball fills.

Why: Linear shape equals a constant rate; that is the interpretation being assessed.

Also acceptable: Accept 'the mass goes up by about 1.2 g every 10 strokes.'

- Describes a straight, rising line
- Connects the straight line to a constant amount of air per stroke

4. Predict the mass of the ball after 100 strokes if the pattern continues. Show the calculation. Then give one reason the real reading might be a little lower than your prediction.

Answer: Predicted mass: $598.4 \text{ g} + (100 \text{ strokes} \times 0.117 \text{ g/stroke}) = 598.4 \text{ g} + 11.7 \text{ g} = 610.1 \text{ g}$. The real reading might be lower because as the ball gets harder, each pump stroke pushes in slightly less air (the pump has to work against higher pressure), or because some air leaks past the needle.

Why: The prediction extends the linear pattern. The second part checks that students understand a pattern can bend outside the measured range.

Also acceptable: Accept 610.4 g if the student used 0.12 g/stroke. Any physically sensible reason for a lower reading earns credit.

5. The ball looked exactly the same before and after pumping. Using the numbers, explain what this dataset proves about air, and what it suggests about the particles air is made of.

Answer: The ball gained 5.85 g on average even though nothing visible was added and its size did not change, so the air pumped in must be matter that has mass. Because the mass rose in equal steps of about 0.12 g per stroke, each stroke must deliver about the same number of tiny particles; mass that comes in countable, repeatable amounts is what we expect if air is made of many identical, extremely small particles.

Why: Full credit needs the 'air has mass' conclusion supported by numbers and a particle inference drawn from the regularity. The particle inference is a suggestion, not a proof, and the student should use language like 'suggests.'

Also acceptable: A student who says 'it cannot be dust or water because the gain was the same in both trials' has given a valid extra argument.

- States that air is matter with mass, citing a number
- Notes the size did not change, so the gain is the added air
- Infers something about particles from the steady pattern

Part B: Testing the halfway rule on four triads

In 1829 Döbereiner noticed that in some sets of three similar elements, the middle element's atomic mass is close to the average of the outer two. The table gives real, rounded atomic masses for four of his triads. Test the rule yourself.

Four triads (real rounded atomic masses, in daltons)

Triad	Lightest element	Middle element	Heaviest element
Metals that react with water	Lithium 7	Sodium 23	Potassium 39
Reactive gases and solids that form salts	Chlorine 35.5	Bromine 80	Iodine 127
Metals in chalky minerals	Calcium 40	Strontium 88	Barium 137
Yellow, red and gray nonmetals	Sulfur 32	Selenium 79	Tellurium 128

6. Complete the table. For each triad, average the lightest and heaviest masses, write the real middle mass, and subtract. Show one full calculation below the table.

Answer: Li, Na, K: $(7 + 39) \div 2 = 23$; actual 23; difference 0. Cl, Br, I: $(35.5 + 127) \div 2 = 81.25$; actual 80; difference +1.25. Ca, Sr, Ba: $(40 + 137) \div 2 = 88.5$; actual 88; difference +0.5. S, Se, Te: $(32 + 128) \div 2 = 80$; actual 79; difference +1.

Why: Each average is the sum of the outer masses divided by 2. All four differences are small compared with the masses themselves (all within about 1.5 Da).

Also acceptable: Accept differences written as absolute values (1.25, 0.5, 1) or as negatives if the student subtracted in the other order and says so.

7. Which triad fits the halfway rule best, and which fits worst? Use the differences from your table as evidence.

Answer: Best: Li, Na, K (difference 0 Da). Worst: Cl, Br, I (difference 1.25 Da).

Why: The student must rank using the computed differences.

Also acceptable: If the student rounded 81.25 to 81, the worst is still Cl, Br, I (difference 1), tied with S, Se, Te; accept either as worst with that explanation.

- Correct best and worst
- Cites the difference values

8. Suppose bromine had not been discovered yet. Using the halfway rule, what mass would you predict for the missing element between chlorine and iodine, and how close would the prediction be to bromine's real 80 Da?

Answer: Predicted mass: $(35.5 + 127) \div 2 = 81.25$ Da, about 81 Da. Bromine's real mass is 80 Da, so the prediction is about 1 Da too high, which is within about 2 percent.

Why: This mirrors Mendeleev's method. Students should recognize the prediction as close but not exact.

Also acceptable: Accept 'about 81 Da, off by about 1 Da.'

9. Look at both parts of this sheet. Part A shows that mass rises by a steady amount with each stroke. Part B shows that masses rise in a fairly regular way down a column. Explain why finding **regular patterns** in numbers was so important for scientists who could not see atoms.

Answer: Scientists could not look at atoms, so numbers were their only window. A regular pattern (equal steps of mass per stroke; middle masses near the average of their neighbors) means there is an underlying rule, and a rule lets you predict what you have not yet measured: the mass after 100 strokes, or the mass of an undiscovered element. Patterns turned measurements into evidence that matter is made of particles and into a tool for finding new elements.

Why: The answer should link regularity to (a) an underlying rule and (b) the power to predict. Referring to both parts of the sheet earns the top marks.

Also acceptable: A student who focuses on Mendeleev's gaps as the payoff of patterns is giving a strong answer.

- Explains that patterns reveal a rule without needing to see atoms
- Explains that rules allow prediction
- Refers to both datasets

Teacher notes

Part A is simulated; the 0.12 g per stroke figure is realistic for a small hand pump (roughly 0.1 L of room air per stroke at 1.2 g per liter). The rigid ball keeps the volume constant so buoyancy does not change; if students ask why that matters, say a bigger balloon is pushed up more by the surrounding air. Part B uses real rounded masses; chlorine is kept at 35.5 on purpose. Answers to q6 should be checked against the key's four lines; the most common error is forgetting to divide by 2. The extension yields about 2.6×10^{21} nitrogen particles per stroke ($0.12 \text{ g} \div (28 \times 1.66 \times 10^{-24} \text{ g}) \approx 2.6 \times 10^{21}$).

Scaffold offered to students: Averaging in two steps

To average two numbers, add them and divide by 2. Write the addition on one line and the division on the next line so that you can check each step. Example with made-up numbers: $12 + 30 = 42$; $42 \div 2 = 21$. For the per-stroke calculation in q2, remember that you are dividing a mass (in grams) by a number of strokes, so the unit of your answer is grams per stroke.

Extension: How many atoms is that?

Air is mostly nitrogen particles, each made of two nitrogen atoms (atomic mass 14 Da each), so one nitrogen particle has a mass of about 28 Da. One dalton is about $1.66 \times 10^{-24} \text{ g}$. Estimate how many nitrogen particles are in the 0.12 g that one pump stroke adds, treating the air as if it were all nitrogen. Write your answer in scientific notation and explain in one sentence why this number supports the idea that atoms are on a completely different scale from everyday objects.