

# Counting Atoms in Fragments: Why a Crystal Has a Ratio, Not a Count

**Learning target:** I can graph and calculate from atom counts to show that an extended structure keeps a constant ratio at any size, and contrast that with the exact atom count of a molecule.

Data and graphing

About 45 min

Individual

MS-PS1-1 · practice

data analysis

independent practice

## Standards connections

**MS-PS1-1** (practice, the atomic-composition idea for extended structures and molecules): Students analyze atom counts to discover why the composition of an extended structure is written as a ratio and the composition of a molecule as an exact count. This is the quantitative reasoning behind the labels on the models the PE asks for, practiced with data rather than by drawing.

**Purpose:** practice · **Time:** about 45 minutes · **Format:** individual

## Answers with reasoning

### Part A: Fragments of a quartz crystal

Quartz is an extended structure of silicon and oxygen atoms. Five teams each cut a different-sized fragment from the same computer model and recorded the program's atom count. The cuts were made along straight lines through the model, so each fragment's surface contains a few partly attached atoms.

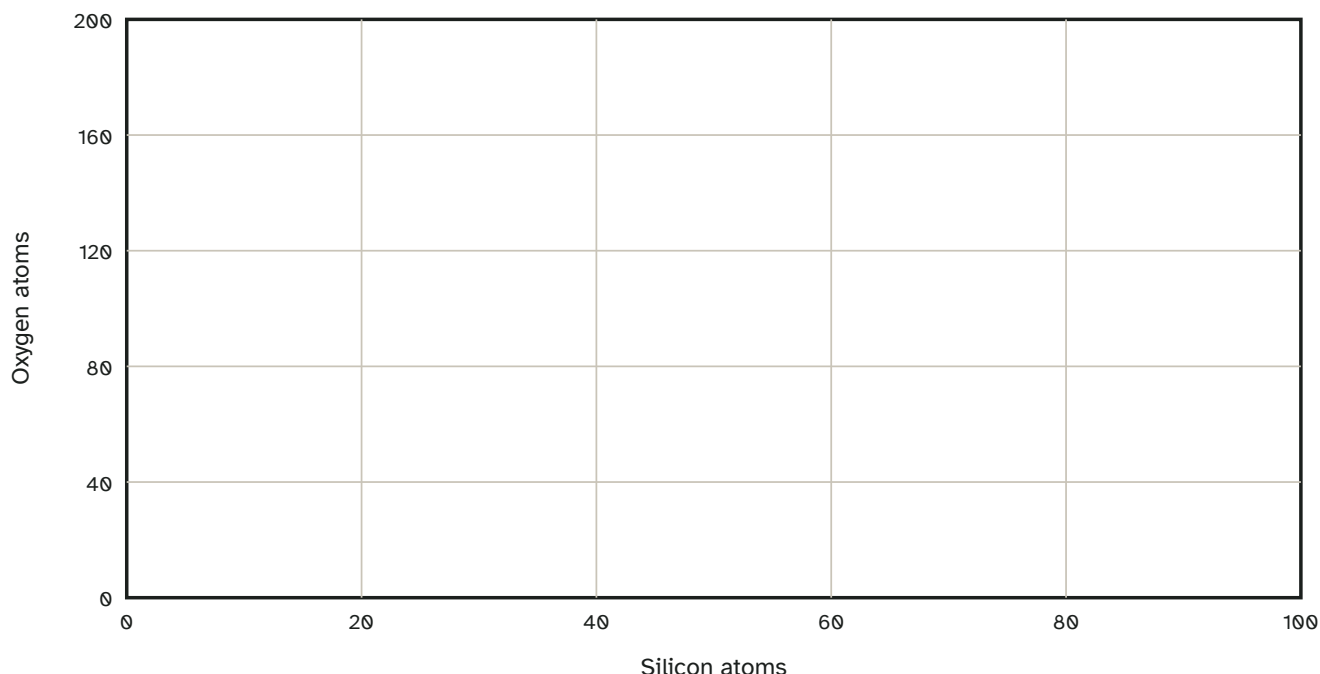
#### Atom counts in five fragments cut from a quartz model (teaching dataset)

*Simulated teaching dataset: the numbers are invented to be realistic, not measured.*

| Fragment | Silicon atoms | Oxygen atoms |
|----------|---------------|--------------|
| A        | 10            | 21           |
| B        | 25            | 50           |
| C        | 40            | 79           |
| D        | 60            | 121          |
| E        | 100           | 200          |

Simulated data for teaching. Realistic for a silicon dioxide model with cut edges.

## Oxygen atoms versus silicon atoms in each fragment



- Complete the table. For each fragment, divide the oxygen count by the silicon count. Show one full division below the table.

**Answer:** A:  $21 \div 10 = 2.10$ . B:  $50 \div 25 = 2.00$ . C:  $79 \div 40 = 1.98$  (1.975 rounds to 1.98). D:  $121 \div 60 = 2.02$  (2.0167). E:  $200 \div 100 = 2.00$ .

**Why:** Each ratio is oxygen divided by silicon. All five are within about 0.1 of 2, and the largest fragment is exactly 2.

*Also acceptable: Accept 1.97 or 1.98 for C and 2.02 for D depending on rounding method.*

- Describe the shape of your graph and where the line meets the left edge. What does this shape tell you about the relationship between the two counts?

**Answer:** The points lie almost exactly on a straight line that rises to the right and passes through (or very near) the origin, so the line meets the left edge at 0 oxygen atoms. A straight line through the origin means the two counts are proportional: every silicon atom comes with about two oxygen atoms, no matter how big the fragment is.

**Why:** Proportionality is the interpretation being assessed; the through-the-origin feature is what distinguishes a constant ratio from a mere trend.

*Also acceptable: Accept 'oxygen is always double the silicon' as the interpretation.*

- Describes a straight rising line
- Notes it passes through or near (0, 0)
- Interprets this as a constant ratio of about 2

3. A sixth team cuts a fragment with 150 silicon atoms. Predict its oxygen count and its total atom count. Show your work.

**Answer:** Oxygen:  $150 \text{ silicon} \times 2 = 300 \text{ oxygen atoms}$ . Total:  $150 + 300 = 450 \text{ atoms}$ .

**Why:** Uses the ratio of 2 oxygen per silicon from the table or the line.

*Also acceptable: Accept a prediction read from an extended graph line of roughly 300 (295 to 305).*

4. A real grain of quartz contains far more atoms than any model. If a grain holds 2,000,000 oxygen atoms, how many silicon atoms does it hold? Show your work and explain which number from your table you used.

**Answer:** Silicon =  $2,000,000 \div 2 = 1,000,000 \text{ silicon atoms}$ , using the ratio of 2 oxygen atoms per silicon atom (the ratio from fragments B and E, or the average of the table).

**Why:** Dividing by the ratio reverses the multiplication in q3. The student should name the ratio used.

*Also acceptable: Accept any answer that uses a ratio between 1.98 and 2.02 with the arithmetic shown (990,000 to 1,010,000).*

5. Fragments A, C and D give ratios that are not exactly 2.00. Using the description of how the fragments were cut, suggest why. Then use the table to say which fragment is farthest from 2.00 and why that makes sense.

**Answer:** The fragments were cut along straight lines, so atoms at the cut surface are partly attached: a cut can leave an extra oxygen hanging on one side or slice one off, giving a count slightly above or below 2. Fragment A, the smallest, is farthest from 2.00 (2.10, off by 0.10, compared with 0.02 for C and D and 0 for B and E). That makes sense because the cut surface holds a larger share of a small fragment's atoms, so the edge errors matter more; in the big fragment E the surface atoms are a tiny share and the ratio comes out exactly 2.00.

**Why:** The surface-to-volume idea explains why the ratio converges as size grows. Fragment B happens to be exactly 2.00 in this dataset; the trend from A to E, not a perfect pattern, is the point.

*Also acceptable: Accept 'the small fragments have more edge compared with their size' in any wording.*

- Attributes the deviation to atoms at the cut surface
- Identifies A as farthest from 2.00 and gives the surface-share (small fragment, more edge) reason

6. The fragments contain 31, 75, 119, 181 and 300 atoms, yet a scientist describes all of them with the same formula,  $\text{SiO}_2$ . Explain what the formula of an extended structure describes, and why there is no such thing as 'the number of atoms in quartz.'

**Answer:** The formula of an extended structure gives the ratio of its elements, one silicon for every two oxygens, not the number of atoms in a particle. Quartz has no separate particle: the pattern of silicon and oxygen repeats until the crystal ends, so a fragment can have 31 atoms or 300 or billions and still be quartz. Asking for 'the number of atoms in quartz' is like asking for the number of tiles in a tile pattern; it depends on where you stop.

**Why:** Full credit requires both the ratio meaning of the formula and the no-fixed-size reason.

*Also acceptable: A student who says the formula gives the 'repeating unit' is correct in spirit; press for the ratio idea.*

- States that  $\text{SiO}_2$  is a ratio
- Explains that the repeating pattern has no fixed size or particle
- Uses the fragment totals or the pattern comparison as support

## Part B: Molecules have exact counts

A molecule is a particle with an edge and an exact number of atoms. The table gives real formulas for eight molecules. In a formula, the small number after a symbol counts that element's atoms in one molecule; no number means one.

### Eight molecules and their atom counts (real)

| Substance                  | Formula                                                      | Atoms of each element in one molecule |
|----------------------------|--------------------------------------------------------------|---------------------------------------|
| Water                      | H <sub>2</sub> O                                             | 2 H, 1 O                              |
| Ammonia (in some cleaners) | NH <sub>3</sub>                                              | 1 N, 3 H                              |
| Methane (natural gas)      | CH <sub>4</sub>                                              | 1 C, 4 H                              |
| Methanol (a fuel)          | CH <sub>3</sub> O                                            | 1 C, 4 H, 1 O                         |
| Carbon dioxide             | CO <sub>2</sub>                                              | 1 C, 2 O                              |
| Vitamin C                  | C <sub>6</sub> H <sub>8</sub> O <sub>6</sub>                 | 6 C, 8 H, 6 O                         |
| Caffeine                   | C <sub>8</sub> H <sub>10</sub> N <sub>4</sub> O <sub>2</sub> | 8 C, 10 H, 4 N, 2 O                   |
| Sucrose (table sugar)      | C <sub>12</sub> H <sub>22</sub> O <sub>11</sub>              | 12 C, 22 H, 11 O                      |

7. Complete the table by adding up the atoms in one molecule of each substance. Show one addition below the table.

**Answer:** Methanol:  $1 + 4 + 1 = 6$  atoms. Vitamin C:  $6 + 8 + 6 = 20$  atoms. Caffeine:  $8 + 10 + 4 + 2 = 24$  atoms. Sucrose:  $12 + 22 + 11 = 45$  atoms.

**Why:** Simple addition of the subscripts; a missing 'no number means one' leads to 5 for methanol, the most common error.

*Also acceptable: None; the counts are exact.*

8. In vitamin C, what is the ratio of hydrogen atoms to carbon atoms, written in lowest terms? Could a chemist who knew **only** that ratio say how many atoms are in one vitamin C molecule? Explain.

**Answer:** Hydrogen to carbon in vitamin C is 8:6, which is 4:3 in lowest terms. No: a 4:3 ratio alone fits C<sub>3</sub>H<sub>4</sub>, C<sub>6</sub>H<sub>8</sub>, C<sub>9</sub>H<sub>12</sub> and so on, so it cannot tell you the molecule has 6 carbons and 8 hydrogens. A molecule needs an exact count; a ratio is not enough.

**Why:** This is the reverse of Part A: for molecules a ratio loses information, because several different molecules can share it.

*Also acceptable: Accept 'it could be any multiple of 3 carbons and 4 hydrogens.'*

9. A classmate says: 'Quartz has a silicon-to-oxygen ratio of 1:2 and water has a hydrogen-to-oxygen ratio of 2:1, so water must be an extended structure too.' Use the ideas of **count** and **particle edge** to explain the mistake.

**Answer:** Water is not an extended structure because each water molecule is a separate particle with an edge, containing exactly 2 hydrogen and 1 oxygen. The 2:1 ratio is simply the count in one particle. Quartz has no particle edge: there is no 'one quartz' with exactly 1 silicon and 2 oxygens, only a pattern, so its formula can only be a ratio. Having a ratio does not make something an extended structure; having no fixed count does.

**Why:** The decisive feature is whether there is a particle with an exact count, not whether a ratio can be written.

*Also acceptable: Accept the contrast 'count versus ratio' in the student's own words if the particle idea is present.*

- Says water molecules are particles with an exact count of 3 atoms
- Says quartz has no particle with a fixed count
- Concludes that the ratio alone does not decide

10. Look at both parts. Part A needed a graph and five fragments to describe quartz; Part B needed only one row per substance. Explain why describing a molecule's composition takes a count while describing an extended structure's composition takes a ratio, and what each description leaves out.

**Answer:** A molecule is a particle with an edge, so its composition is a count: one row of the table says exactly which atoms and how many (sucrose: 12 C, 22 H, 11 O). An extended structure has no edge and no fixed size, so the only thing that stays the same from fragment to fragment is the proportion of its elements; that is why Part A needed several fragments and a graph to show that the ratio is constant. A count leaves out how the atoms are arranged (butane and isobutane share a count). A ratio leaves out the total number of atoms and the size of the piece, and also the arrangement.

**Why:** Full credit requires the count-versus-ratio contrast tied to the particle-edge reason and a statement of what each description omits.

*Also acceptable: A student who notes that the ratio of a molecule can also be written but is not enough to identify it (from q8) is extending the idea well.*

- Molecule: exact count because the particle has an edge
- Extended structure: ratio because size is not fixed
- Names at least one thing each description leaves out (arrangement, total, size)

## Teacher notes

Part A is simulated: the fragment counts were chosen so that larger fragments sit closer to the 1:2 ratio, mimicking the surface effect of cutting a real lattice; no real quartz fragment was counted. Part B uses real formulas. The most common errors are forgetting that 'no number means one' (methanol, water) and dividing silicon by oxygen instead of oxygen by silicon in q1, which gives 0.5; both are worth a whole-class minute. The textbook's Figure 3A caption on page 26 says quartz has one oxygen per silicon; that is a misprint, and the main text on the same page correctly gives 1:2 (silicon dioxide,  $\text{SiO}_2$ ). Extension answers: each silicon claims 4 oxygens but each oxygen is shared by 2 silicons, so oxygens per silicon =  $4 \div 2 = 2$ , giving  $\text{SiO}_2$ ; the imaginary crystal gives  $6 \div 3 = 2$  as well, so its formula would also be 1:2; a cut edge leaves oxygens attached to only one silicon, so they are no longer shared and the count drifts.

### **Scaffold offered to students: Reading a ratio from a graph**

When a straight line passes through the point (0, 0), the ratio of the two quantities is the same at every point on the line. To find it, pick any point on the line, divide the y-value by the x-value, and you have the number of oxygen atoms per silicon atom. Check it with fragment E:  $200 \div 100 = 2$ . For q3 and q4, use that ratio as a multiplier (oxygen =  $2 \times$  silicon) or a divider (silicon = oxygen  $\div 2$ ).

### **Extension: Where the 1:2 ratio comes from**

In quartz every silicon atom is bonded to 4 oxygen atoms, and every oxygen atom is bonded to 2 silicon atoms, so each oxygen is shared between two silicons. Show with a calculation that this sharing forces a ratio of exactly 1 silicon to 2 oxygen. Then imagine a different crystal in which every metal atom is bonded to 6 oxygen atoms and every oxygen is shared by 3 metal atoms; find its ratio. Finally, explain why a cut edge breaks the sharing and makes small fragments drift away from the exact ratio.