

Design Challenge: A Float-Sink Line for Five Plastics

Learning target: I can use density data to design and justify a sequence of liquid tanks that separates mixed plastic flakes, evaluate competing designs against criteria and constraints, and recognize when density alone cannot finish the job.

Transfer and design

About 50 min

Pairs

MS-ETS1-2 · practice

MS-ETS1-3 · practice

MS-PS1-2 · practice

engineering design

data analysis

writing

Standards connections

MS-ETS1-2 (practice): Students evaluate three competing tank designs with a systematic table against stated criteria (pure streams, cost, safety) and constraints (three tanks, available liquids), then build and justify their own. This practices the PE's systematic evaluation without a full design cycle.

MS-ETS1-3 (practice): Students analyze what each candidate design does well and poorly and are asked to combine the best features into a new design that better meets the criteria, which is the heart of this PE.

MS-PS1-2 (practice): The whole design rests on interpreting density as an identifying property of each plastic and on recognizing that two substances with nearly equal densities need a different property. No chemical reaction is involved.

Purpose: extend · **Time:** about 50 minutes · **Format:** pairs

Answers with reasoning

The brief from Second Life Plastics — 'We buy bales of shredded flakes that mix five plastics: polypropylene (PP), high-density polyethylene (HDPE), polystyrene (PS), PET and PVC. Buyers pay well for flakes that are one plastic only and refuse flakes that are mixed. Our building has room for at most **three tanks**. Each tank holds one liquid; flakes that float are skimmed off the top and flakes that sink are collected from the bottom, so a tank splits a stream into two. Design the line: which liquid goes in each tank, what order the streams flow, and which plastics come out of each exit. Tell us honestly what the line cannot do.'

Data for the design (real rounded values)

Item	Density (g/cm ³)	Notes
Polypropylene (PP) flakes	0.90	caps, yogurt tubs
High-density polyethylene (HDPE) flakes	0.95	milk jugs
Polystyrene (PS) flakes	1.05	clear cups
PET flakes	1.38	soda bottles
PVC flakes	1.40	blister packs, pipe
Plain water	1.00	cheapest; safe
Salt water	adjustable 1.02 to 1.20	cheap; safe; salt must be topped up
Water-and-alcohol mix	adjustable 0.92 to 0.99	costly; flammable; needs ventilation

Part A: Do the math before you design

1. Write 'floats' or 'sinks' in every box. A flake floats when its density is lower than the liquid's.

Answer: Plain water (1.00): PP floats, HDPE floats, PS sinks, PET sinks, PVC sinks. Salt water at 1.15: PP floats, HDPE floats, PS floats, PET sinks, PVC sinks. Alcohol mix at 0.93: PP floats, HDPE sinks, PS sinks, PET sinks, PVC sinks.

Why: A flake floats when its density is below the liquid's. The three columns give the three cut points the designs use.

Also acceptable: None; the table has one correct filling.

2. Find the density difference between each neighboring pair of plastics (PP-HDPE, HDPE-PS, PS-PET, PET-PVC). Which pair is closest, and which liquid on the list could be set between them?

Answer: PP-HDPE: $0.95 - 0.90 = 0.05$. HDPE-PS: $1.05 - 0.95 = 0.10$. PS-PET: $1.38 - 1.05 = 0.33$. PET-PVC: $1.40 - 1.38 = 0.02$. The closest pair is PET-PVC at 0.02 g/cm^3 ; no liquid on the list can be set between 1.38 and 1.40 (salt water stops at 1.20), and even if one could, a 0.02 margin is too small to work reliably. The PP-HDPE gap of 0.05 can be split by the alcohol mix set between 0.90 and 0.95 , for example 0.93 .

Why: The differences show where density separation is easy, hard and impossible.

Also acceptable: Accept 'about 0.92 to 0.94' for the alcohol setting.

Part B: Define the problem precisely

A requirement you cannot test is not a requirement. 'Sort the plastics well' cannot be tested. 'Every exit stream contains one plastic only' can be tested by sampling flakes at each exit. Write your criteria (what a good design achieves) and constraints (limits the design must obey) so that each one could be checked.

3. Write three criteria and three constraints for the line. For each criterion say how you would measure it. Rank the criteria from most to least important and give a reason for your top choice.

Answer: Sample criteria: (1) Number of exits that hold one plastic only, measured by sampling 100 flakes at each exit and counting; (2) operating cost, measured in dollars of liquid and salt or alcohol used per ton; (3) worker safety, measured by whether any tank holds a flammable liquid and needs ventilation. Sample constraints: at most three tanks; only the listed liquids at the listed densities; each tank splits a stream into exactly two (float and sink). Top criterion: purity (criterion 1), because the brief says buyers refuse mixed flakes, so a cheap line that produces mixed streams earns nothing.

Why: Criteria must be measurable; constraints must come from the brief or the data. The ranking should be defended with the buyer's requirement.

Also acceptable: A student who ranks safety first because a fire would shut the plant down is making a defensible choice; accept with that reason.

- Three testable criteria with a way to measure each
- Three constraints drawn from the brief or data
- Ranking with a reason tied to the brief

4. One pair of plastics cannot be separated by any liquid on the list, no matter how many tanks you had. Name the pair and use numbers to explain why.

Answer: PET (1.38) and PVC (1.40). Their densities differ by only 0.02 g/cm³, and the densest liquid available is salt water at 1.20, so both always sink together. Even a liquid at 1.39 would be unreliable because the margin on each side is 0.01, smaller than ordinary variation in flakes.

Why: The brief asked for honesty about what the line cannot do; this is the main limit.

Also acceptable: Accept an answer that also notes the PP-HDPE pair is hard (0.05) but possible.

- Names PET and PVC
- Uses the 0.02 difference and the 1.20 ceiling

Part C: Evaluate three proposed designs

Three candidate designs

1 Design 1: Water first, then split both ways

Tank 1: plain water. Floaters go to Tank 2: alcohol mix at 0.93. Sinkers go to Tank 3: salt water at 1.15. Four exits.

2 Design 2: Salt first, cheap all the way

Tank 1: salt water at 1.20. Floaters go to Tank 2: salt water at 1.02. Floaters from Tank 2 go to Tank 3: plain water. Four exits.

3 Design 3: Two tanks only

Tank 1: plain water. Sinkers go to Tank 2: salt water at 1.10. Floaters from Tank 1 go straight to a bin. Three exits, one tank left empty.

5. Trace every plastic through each design using your Part A table and complete the evaluation.

Answer: Design 1: Tank 1 water: PP and HDPE float, PS, PET and PVC sink. Tank 2 (0.93): PP floats, HDPE sinks. Tank 3 (1.15): PS floats, PET and PVC sink. Exits: PP; HDPE; PS; PET+PVC. One-plastic exits: 3. Uses alcohol: yes. Three tanks: yes. Design 2: Tank 1 (1.20): PP, HDPE, PS float; PET and PVC sink. Tank 2 (1.02): PP and HDPE float; PS sinks. Tank 3 (water): PP and HDPE both float. Exits: PET+PVC; PS; PP+HDPE (Tank 3 separates nothing, since no plastic sinks there). One-plastic exits: 1. Uses alcohol: no. Three tanks: yes. Design 3: Tank 1 water: PP and HDPE float to a bin; PS, PET, PVC sink. Tank 2 (1.10): PS floats; PET and PVC sink. Exits: PP+HDPE; PS; PET+PVC. One-plastic exits: 1. Uses alcohol: no. Three tanks: yes (uses two).

Why: Students must trace each plastic. The key error to watch for is in Design 2, where the plain-water tank does nothing because every plastic reaching it is lighter than water.

Also acceptable: If a student counts Design 2's Tank 2 as separating PS from the others, that is right; Tank 3 is the wasted one.

- Exit contents correct for all three designs
- Counts of one-plastic exits: 3, 1, 1
- Alcohol use and tank constraint recorded

6. Which design best meets your top criterion? Which is cheapest and safest? Name one feature from a weaker design that is still worth keeping, and say why.

Answer: Design 1 best meets purity: three one-plastic exits and only PET+PVC left mixed, which no design can fix. Design 3 is cheapest and safest (two tanks, water and salt only, nothing flammable). Worth keeping from Design 3: the plain-water first tank followed by salt water at 1.10 to isolate PS cheaply; from Design 2, the idea of using salt water instead of alcohol wherever a gap is wide enough.

Why: The comparison should refer to the q5 counts and to the cost/safety notes in the data table.

Also acceptable: Accept Design 2 as cheapest if the student notes it costs more salt than Design 3 but is still non-flammable.

- Design 1 named for purity with the count
- Design 3 (or 2) named for cost and safety
- A feature from a weaker design kept with a reason

Part D: Decide and justify

7. Design your own line (you may combine ideas from Part C). For each tank state the liquid and its density, which stream enters, and which plastics float and sink, with the numbers that prove it. Then state what your line cannot do and why. End with one sentence on the trade-off you accepted (cost, safety or purity).

Answer: Model design: Tank 1, plain water (1.00): the mixed stream enters; PP (0.90) and HDPE (0.95) float; PS (1.05), PET (1.38) and PVC (1.40) sink. Tank 2, salt water at about 1.15: the Tank 1 sinkers enter; PS floats ($1.05 < 1.15$) and exits pure; PET and PVC sink together and exit as a mixed stream. Tank 3, alcohol mix at 0.93: the Tank 1 floaters enter; PP floats ($0.90 < 0.93$) and exits pure; HDPE sinks ($0.95 > 0.93$) and exits pure. Result: four exits, three pure (PP, HDPE, PS) and one mixed (PET+PVC). What it cannot do: separate PET from PVC, because their densities differ by 0.02 and no available liquid falls between them. Trade-off: the alcohol tank raises cost and adds a flammable liquid, accepted because without it PP and HDPE would leave mixed and could not be sold.

Why: Full credit requires every tank specified with liquid and density, correct float/sink outcomes with numbers, an honest statement of the PET-PVC limit, and a named trade-off. A student who chooses to skip the alcohol tank and sell PP+HDPE mixed has made a legitimate decision if the trade-off (purity for safety and cost) is stated.

Also acceptable: The order of Tanks 2 and 3 can be swapped; a salt-water setting anywhere from 1.06 to 1.20 works for the PS split (1.10 gives the safest margins). An alcohol setting between 0.91 and 0.94 works for the PP-HDPE split.

- Each tank has a liquid and density
- Float/sink outcomes correct with numbers
- States the PET-PVC limit with the reason
- Names a trade-off and the reason for accepting it

8. Draw your line as a flow diagram: a box for each tank showing the liquid and its density, arrows labeled 'floats' and 'sinks' leaving each tank, and a labeled exit bin at the end of every path naming the plastic or plastics it holds.

Answer: A flow diagram with three tank boxes labeled with the liquid and density, 'floats' and 'sinks' arrows leaving each, and exit bins labeled PP, HDPE, PS and PET+PVC (or the student's own design's exits), matching the design in q7.

Why: The diagram must agree with q7; mismatches between the written design and the drawing are the most common error.

Also acceptable: A two-tank diagram is acceptable if q7 chose two tanks.

- Every tank labeled with liquid and density
- Both arrows from every tank labeled
- Every path ends in a labeled bin
- Matches q7

9. Propose one test that uses a property **other than density** to separate the pair from q4, and name one safety or cost concern it raises. Use the data in the notes of the Reading worksheet or your own knowledge, and say what you would need to look up before trusting the test.

Answer: Sample: use heating behavior. PET melts cleanly near 250 °C, while PVC starts to break down near 150 °C; a small sample warmed to about 170 °C would show PVC softening and discoloring while PET stays unchanged. Concern: PVC gives off hydrogen chloride gas when it breaks down, which is corrosive and harmful to breathe, so the test needs a fume hood and would be slow and costly for whole streams. Another sample: a near-infrared sorter that reads each plastic's reflected-light pattern; the concern is cost and that black flakes cannot be read. Before trusting either, look up the exact temperatures or the sorter's error rate for mixed flakes.

Why: The answer should use a non-density identifying property, name a concern, and show the habit of checking a reference before relying on a test.

Also acceptable: Accept solubility in a solvent if the student also names the solvent hazard and the need to look up which solvent.

- Test uses a property other than density
- A safety or cost concern named
- States what must be looked up

Scoring rubric

Criterion	4 • Exemplary	3 • Proficient	2 • Developing	1 • Beginning
Problem definition (Part B)	Three measurable criteria and three constraints from the brief, ranked with a reason tied to the buyer's requirement; the PET-PVC limit explained with numbers.	Criteria and constraints present; one is not measurable or the ranking lacks a reason.	Criteria and constraints mixed up, or fewer than three of either; PET-PVC limit named without numbers.	Vague requirements that could not be tested; limit not identified.
Systematic evaluation (Part C)	All three designs traced correctly; one-plastic exit counts of 3, 1 and 1; strengths of weaker designs identified and a feature kept with a reason.	Designs traced with one error (commonly Design 2's wasted tank); comparison made against criteria.	Designs compared by impression rather than by tracing flakes; counts missing or wrong for two designs.	No systematic comparison.
Design and justification (Part D)	Every tank has a liquid and density; every float/sink outcome is proved with numbers; the PET-PVC limit is stated honestly; a trade-off is named and defended; diagram matches the text.	Design works and is justified, but one outcome lacks its number or the trade-off is not stated.	Design has a tank that separates nothing, or outcomes asserted without numbers.	Design does not follow from the data or violates a constraint without noting it.
Use of the science	Treats density as an identifying property, explains floating and sinking by comparing densities, and proposes a sound non-density property for the PET-PVC pair with a concern and a reference check.	Density reasoning sound; the non-density proposal is vague or lacks a concern.	Uses 'heavier' and 'lighter' for density without volume; non-density proposal missing.	Science errors in the float/sink reasoning.

Teacher notes

Densities are real rounded values: PP 0.90, HDPE 0.95, PS 1.05, PET 1.38, PVC 1.40 g/cm³; salt water can be raised to about 1.20 g/cm³ at saturation; water-alcohol mixtures between 0.92 and 0.99 are realistic for 10 to 50 percent alcohol by mass. Float-sink separation and the PET-PVC overlap are real industry issues (see the Reading worksheet). Design 2 is the deliberate trap: its last tank separates nothing because only floaters reach it. The best achievable result with these liquids is three pure streams plus a mixed PET+PVC stream. Pairs should finish Part A before Part C; groups that skip the tracing table nearly always misjudge Design 2. If students propose burning PVC to identify it, stop that line: PVC releases hydrogen chloride when heated, which is why the q9 key routes it through a fume hood and a reference check. The extension calculation: a 1 cm³ PET flake with 0.3 cm³ of trapped air has mass $0.7 \times 1.38 = 0.97$ g in a total volume of 1.0 cm³, so its effective density is about 0.97 g/cm³ and it would float in plain water; a wetting agent or a brief soak that releases bubbles reduces the error.

Scaffold offered to students: Trace one flake at a time

Pick one plastic, say PS at 1.05, and walk it through a design: in plain water (1.00) it sinks, so it goes wherever the sinkers go; in salt water at 1.15 it floats, so it is skimmed there. Write the exit it reaches. Then do the next plastic. When two plastics reach the same exit, that exit is mixed. Count the exits that hold only one plastic; that number is the design's score on purity.

Extension: Flakes that lie

Real flakes sometimes trap tiny air bubbles, which makes a sinking flake float, and some PET bottles carry HDPE caps and PP labels that shred together. Explain how a trapped bubble changes the effective density of a flake (use a 1 cm³ PET flake with 0.3 cm³ of air as an example and calculate), then propose one change to the line, such as a wetting step or a second pass, that would reduce the error. Say which criterion it improves and which constraint it strains.